Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Metric Space

Subject Code : 4SC05MSC1 **Branch: B.Sc.(Mathematics)**

Semester : 5 **Date : 28/03/2017** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define fixed point of function. (01)
b) What is $int\ A$? where A={1,2,3 }. (01)
c) True/false: Arbitrary intersection of closed set is closed. (01)
d) What is derived set? (01)
e) Does [0,1] become closed set in usual metric ? (01)
f) What is closer of set A? (01)
g) True/false: Every open sphere is an open set (01)
h) What is open sphere in $\boldsymbol{R^3}$? (01)
i) Define : Equivalent metrics. (01)
j) True/false : Continuous image of connected set is connected. (01)
k) Find $\bar{A}$ if $A = [0,2) \cup (2,4]$ (01)
l) Define : Closed set . (01)
m) True/false: Closer of any set is always closed. (01)
n) If A is any open set then what will be $int\ A$? (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

(a) What is metric space? The function d defined by $d(x,y) = |x-y|$ for all $x, y \in R$, Show that (R,d) is metric space. **(07)**

(b) If (X,d) be any metric space. Show that the function d_1 defined by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}, \forall\ x,y \in X$ is a metric on . **(07)**

Q-3 **Attempt all questions** **(14)**

(a) What is an open set? **(07)**
Which of the following sets are open sets.
$(i)\ (-2,2)\ on\ R$
$(ii)\ \{\ (x,y)\ /\ x\ +\ y = 0\ \}\ on\ R^2$
$(iii)\ \{\ (x,y)\ /\ x^2\ +\ y^2 >\ 1\ \}\ on\ R^2.$

(b) Let the set l_∞ of all bounded sequences $\{x_n\}$ of real number with the function d defined by $d(\{x_n\},\{y_n\}) = up\{|x_n - y_n|, n \in N\}$, show that (l_∞, d) is metric space. **(07)**

Q-4 **Attempt all questions** **(14)**

(a) If d_1 and d_2 are two metric space on , then show that $(X,\ d_1 + d_2)$ is metric space but $(X, {d_1}^2)$ is not a metric space. **(07)**

(b) Which of the following sets are closed sets. Explain with figure. **(07)**
$(i)\ [-2,1]\ on\ R$
$(ii)\ \{\ (x,y)\ /\ x\ =\ y\ \}\ on\ R^2$
$(iii)\ \{\ (x,y)\ /\ x\ +\ y\ =\ 1\ \}\ on\ R^2$
$(iv)\ \{\ (x,y)\ /\ x\ +\ y >\ 1\ \}\ on\ R^2,$

Q-5 **Attempt all questions** **(14)**

(a) What is limit point of the set ? find A' for the following sets **(07)**
(1) $(R, d_u),\ A = [0,1]$
(2) $(R, d_u),\ A = (0,1]$
(3) $(R, d_u),\ A = \{1,2,3,4, \ldots\ldots\}.$

(b) Let (X,d) be a metric space and $Y \subseteq X$, then prove that a subset to be open in (Y, d_Y) if and only if there exists a set G open in (X,d) such that $A = G \cap Y$. **(07)**

Q-6 **Attempt all questions** **(14)**

(a) Show that $\overline{A \cup B} = \bar{A} \cup \bar{B}$ but $\overline{A \cap B} \neq \bar{A} \cap \bar{B}$. **(07)**

(b) What is Boundary point of subset A ? Find $intA$, $extA$, bdA for the following. **(07)**
(1) $X = R, A = Q$
(2) $X = R, A = (1,2] \cup (3,4)$

Q-7 **Attempt all questions** **(14)**

(a) If $A \subseteq R$ is closed and bounded ,Show that A is compact. **(07)**

(b) What is compact set? Show that every compact subset F of a metric space (X,d) is closed. **(07)**

Q-8 **Attempt all questions** **(14)**

(a) What is connected set? if Y is connected subset of metric space (X,d), show that Y cannot be expressed as disjoint union of two non-empty closed sets in Y. **(07)**

(b) State and prove Banach fixed point theorem **(07)**